

# Tape diagrams

ACT Adjoint School 2022  
Nathan, Phoebe, Ralph, Rowan

s/o Filippo, Chad, organizers

Motto: "string diagrams for string diagrams"

# Tape diagrams

ACT Adjoint School 2022

Nathan, Phoebe, Ralph, Rowan

s/o Filippo, Chad, organizers

- Intro to tape diagrams
- Euclid's algorithm

# Tape diagrams

- Motivation: Two monoidal products in Rel...

# Tape diagrams

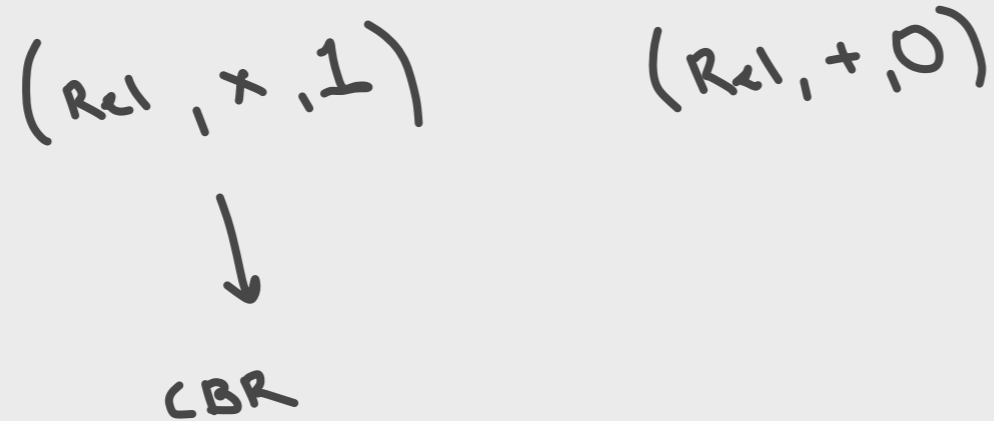
- Motivation: Two monoidal products in Rel...

$(\text{Rel}, \times, \mathbb{1})$

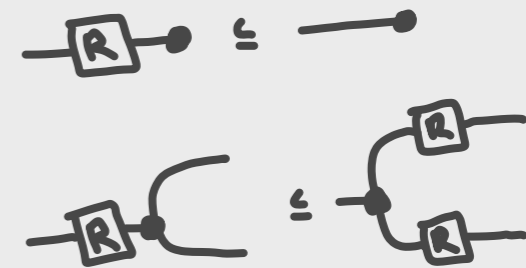
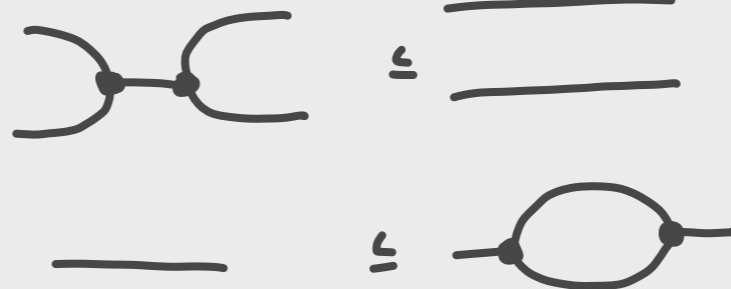
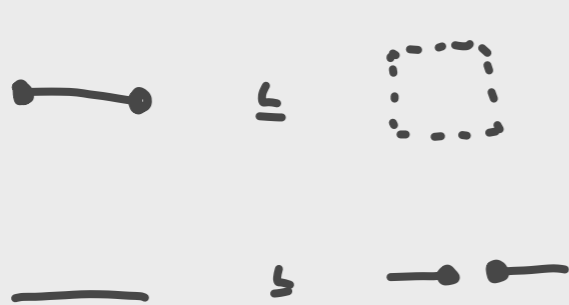
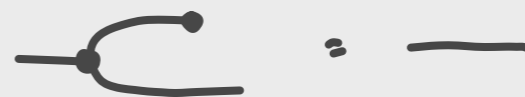
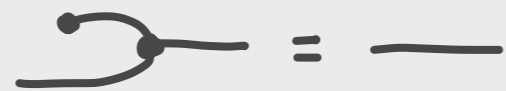
$(\text{Rel}, +, \mathbb{0})$

# Tape diagrams

- Motivation: Two monoidal products in Rel...



# Tape diagrams



# Tape diagrams

- Motivation: Two monoidal products in Rel...

$(\text{Rel}, \times, \mathbb{1})$



CBR

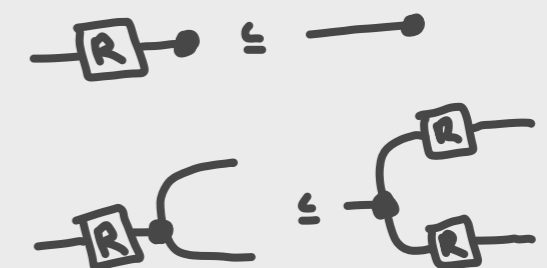
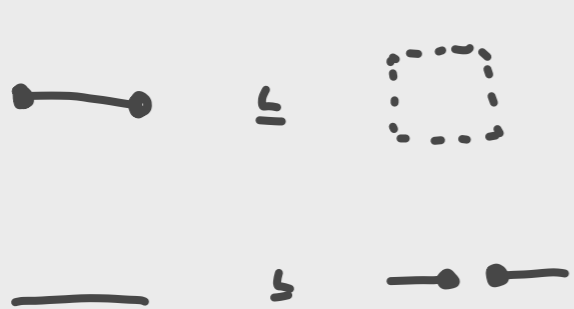
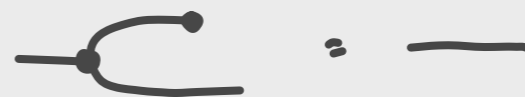
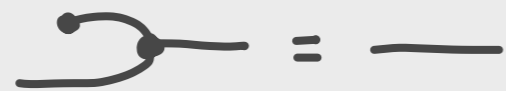
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Finite Biproducts



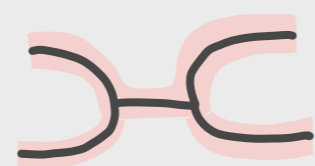
# Tape diagrams



# Tape diagrams



$\varepsilon$



$\varepsilon$



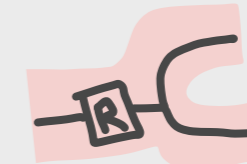
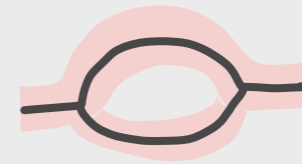
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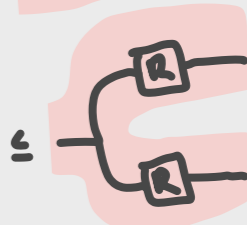
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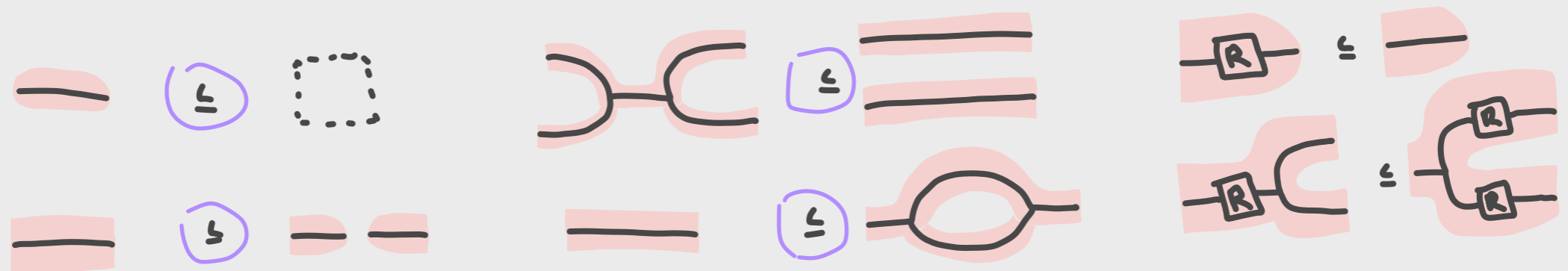
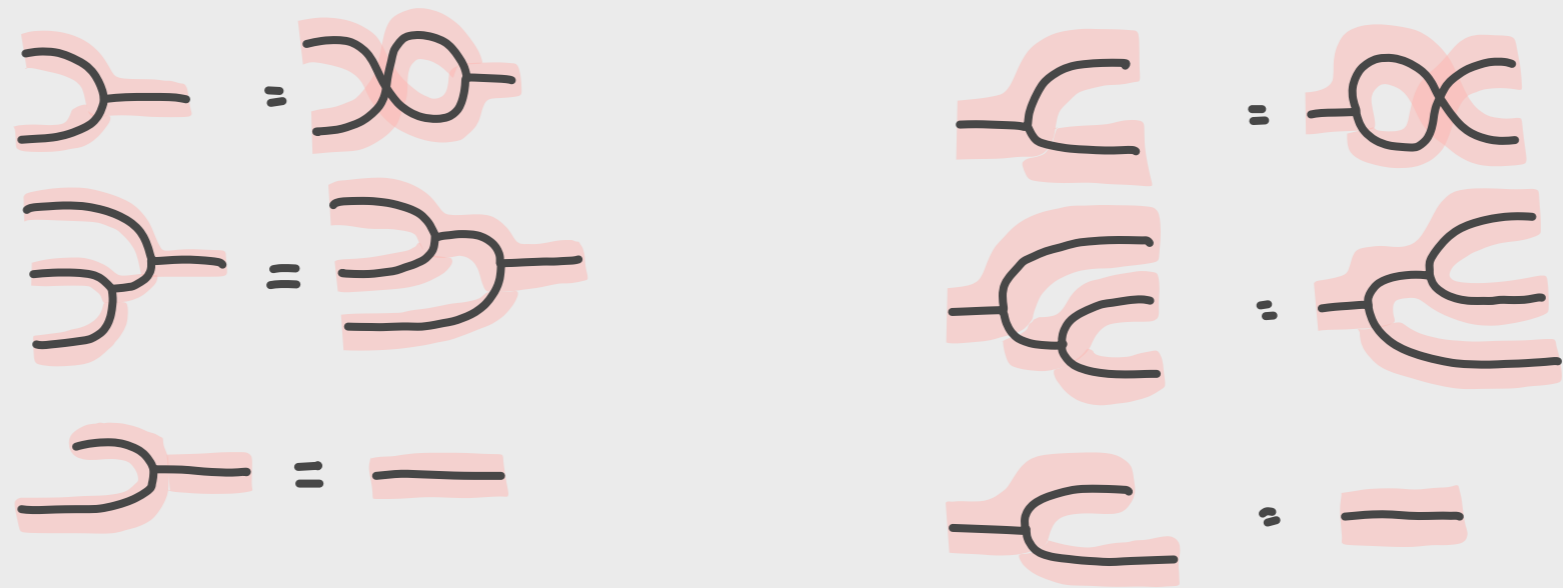
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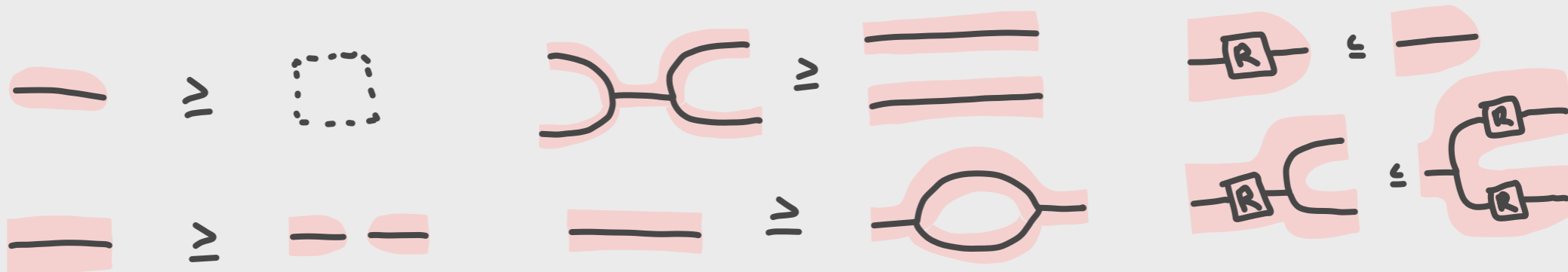
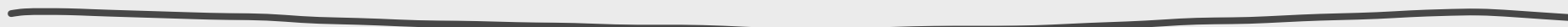
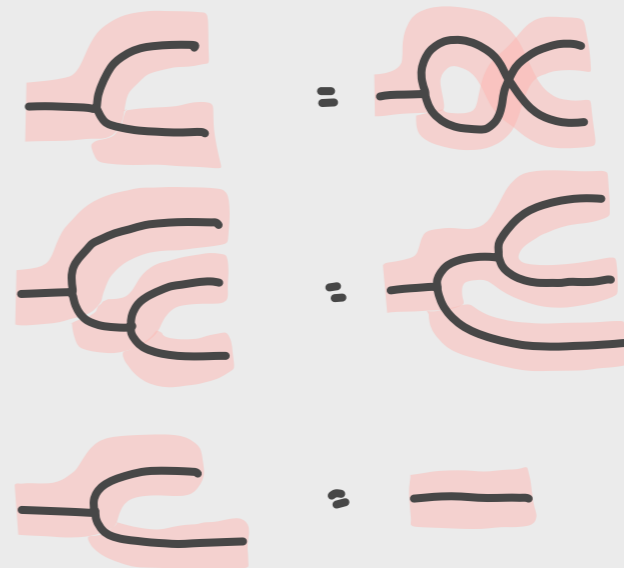
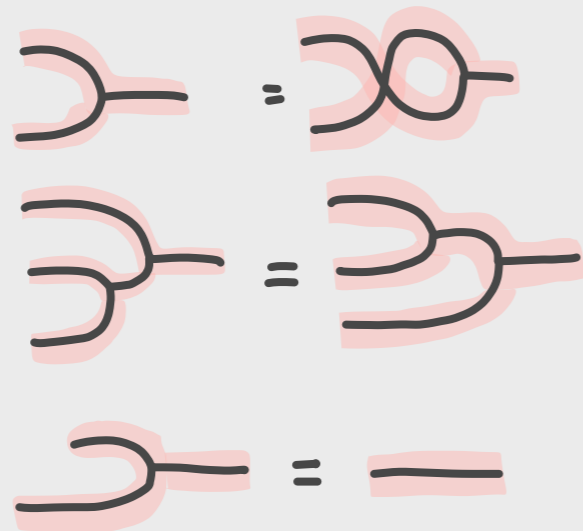
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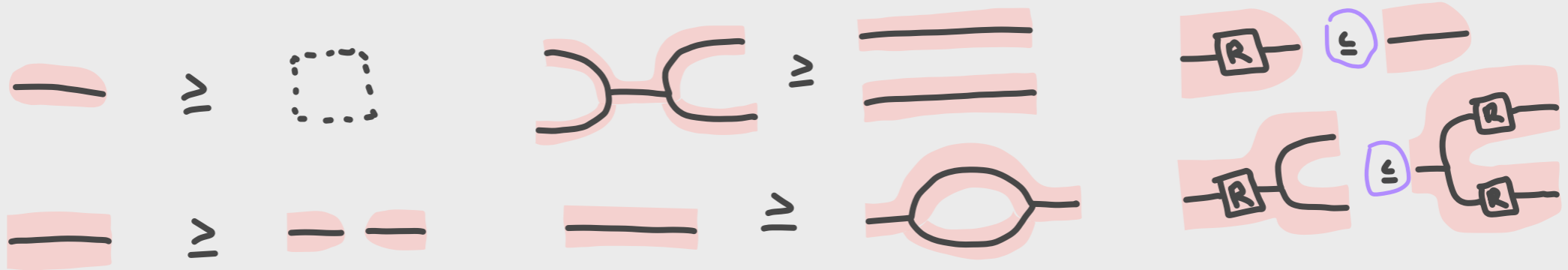
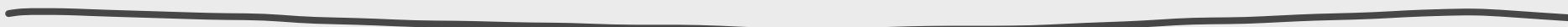
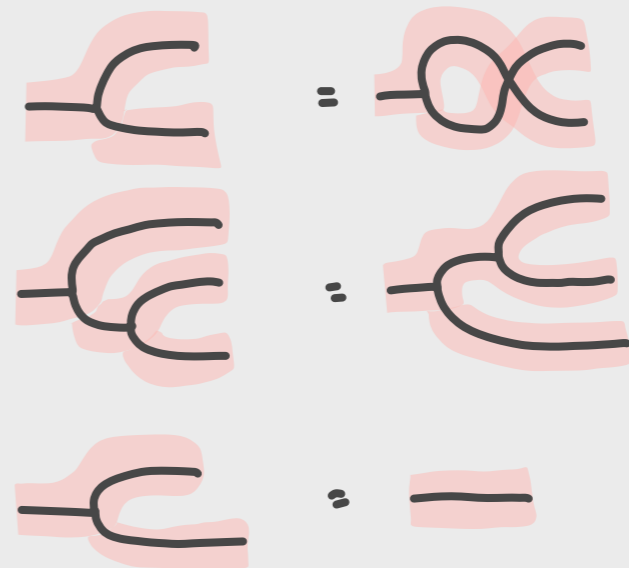
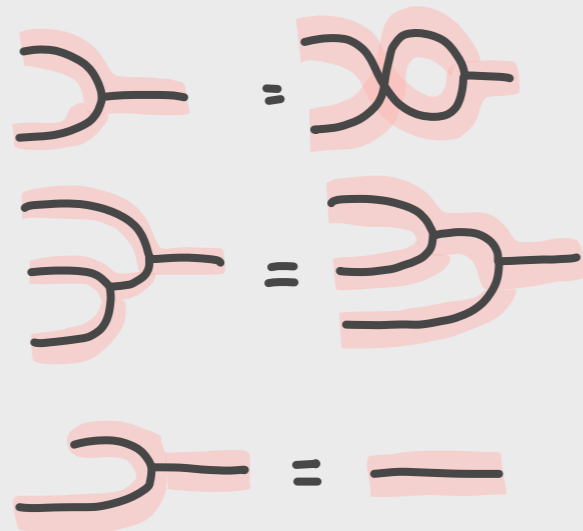
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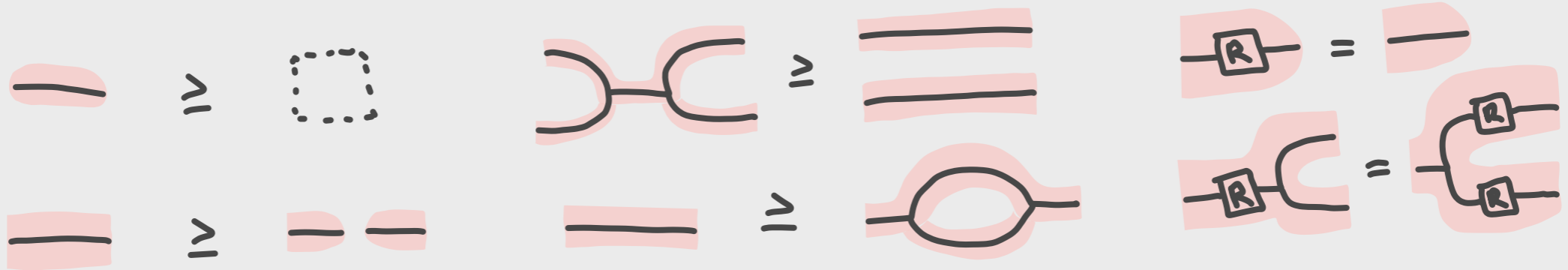
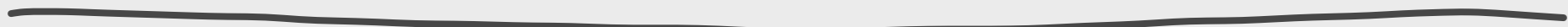
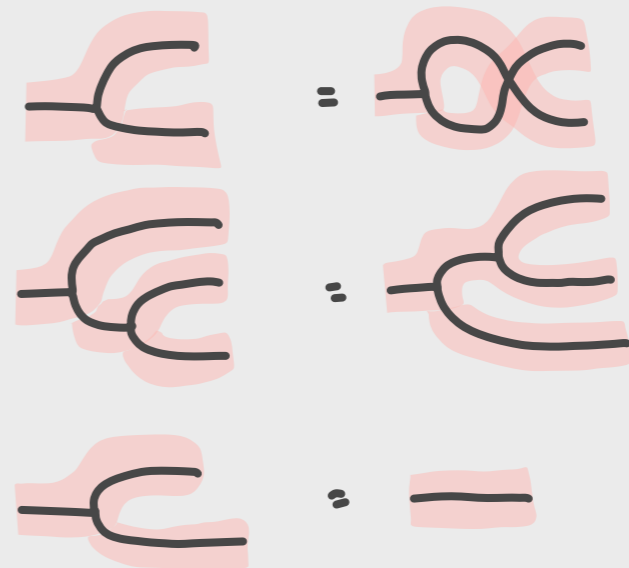
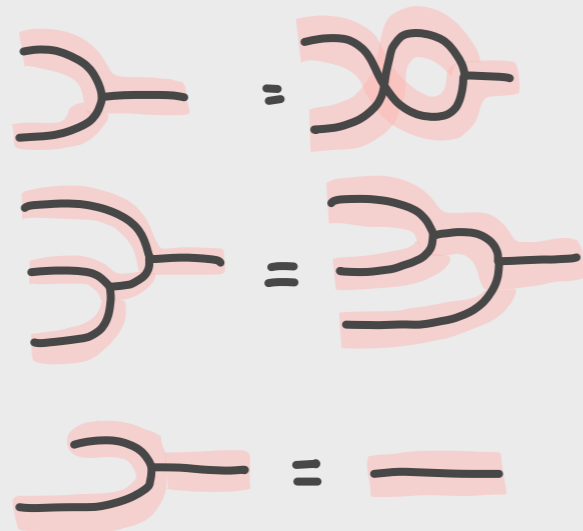
# Tape diagrams



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# Tape diagrams

- Motivation: Two monoidal products in Rel...

$(\text{Rel}, \times, \mathbb{1})$

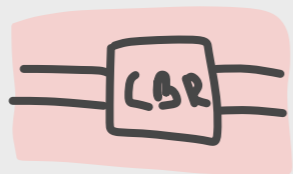


CBR

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Finite Biproducts



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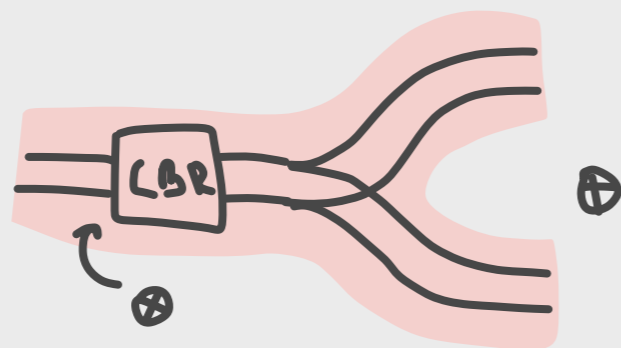


CBR

$(\text{Rel}, +, \mathbb{0})$

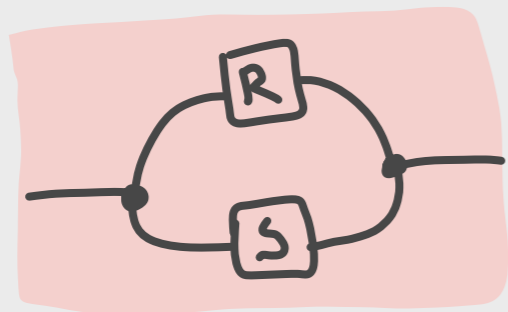


Finite Biproducts

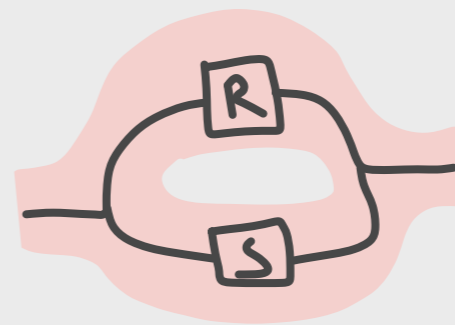




# Tape diagrams



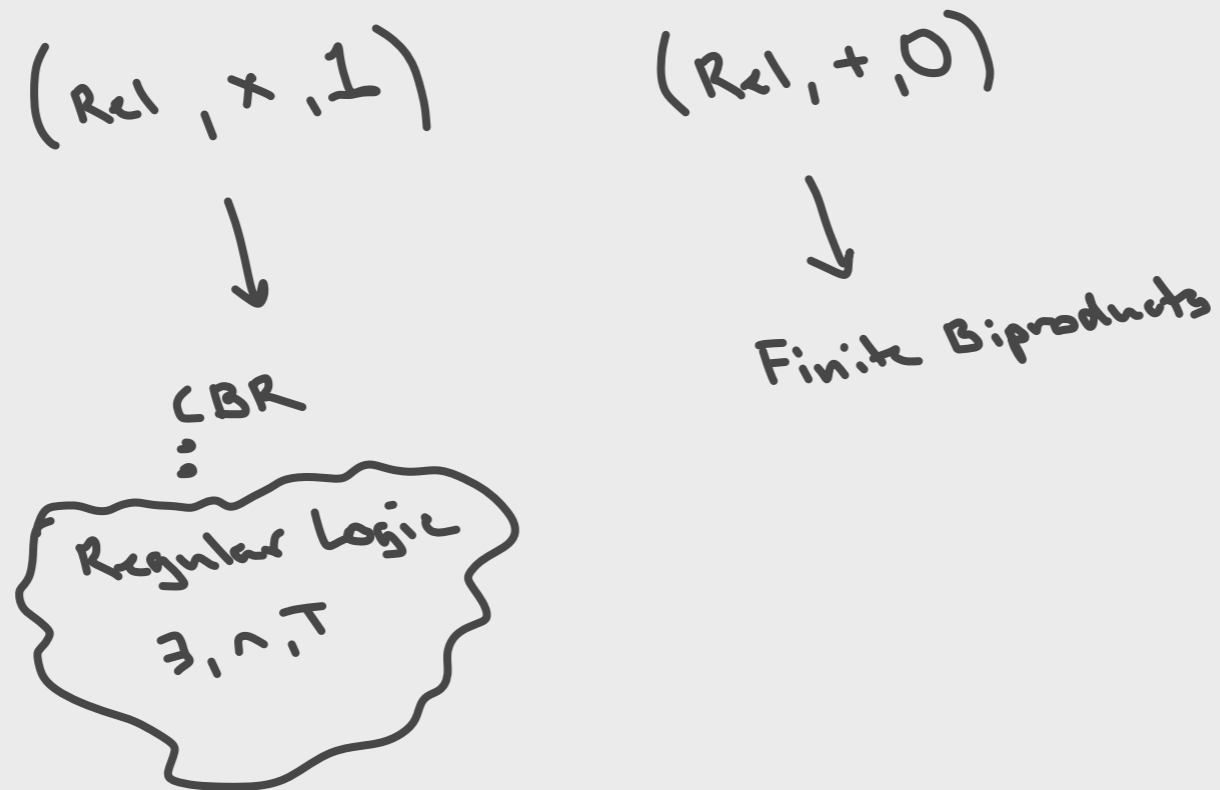
$R \cap S$



$R \cup S$

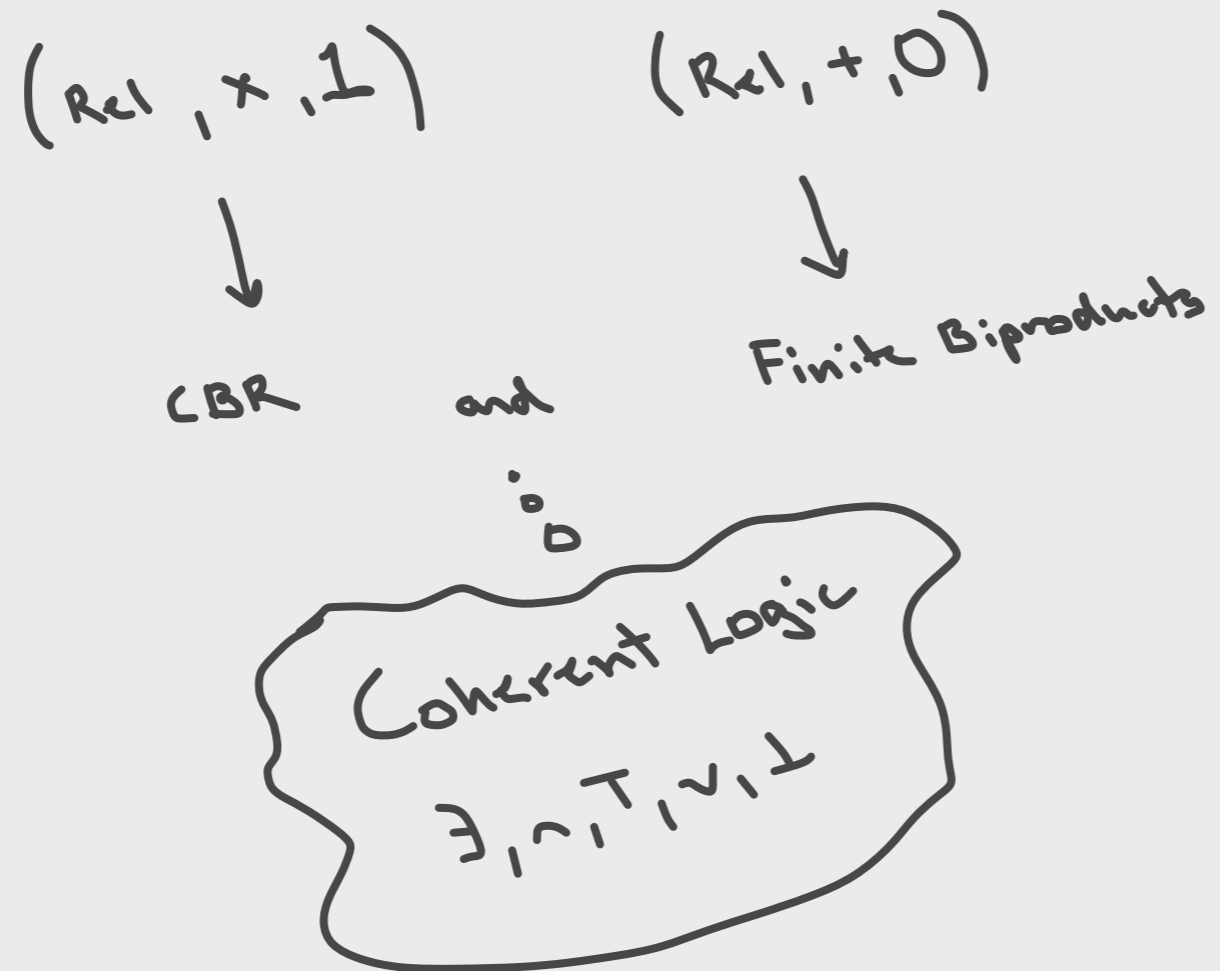
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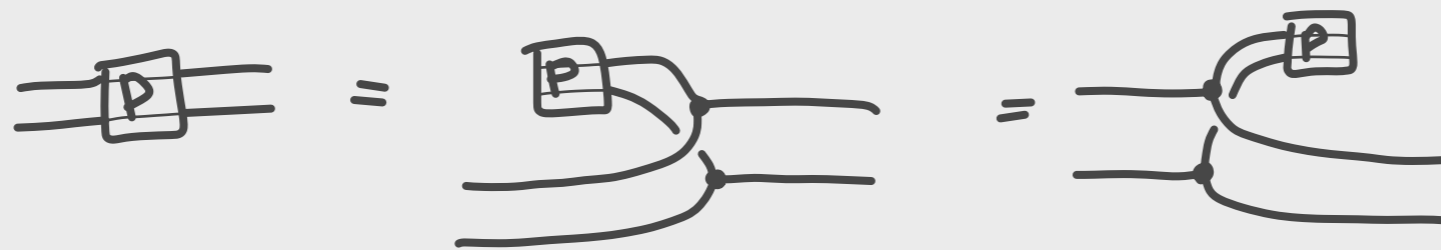
# Tape diagrams

- Motivation: Towards programming...

# Tape diagrams

- Predicates and Co-reflexives

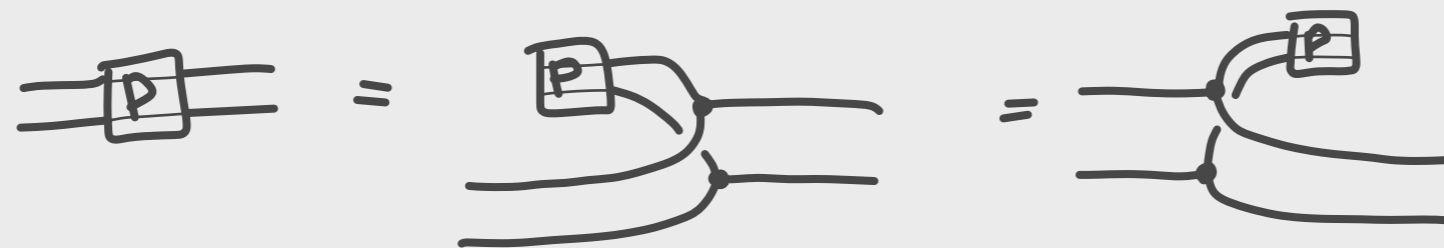
Given a Predicate  $P \in X^2$



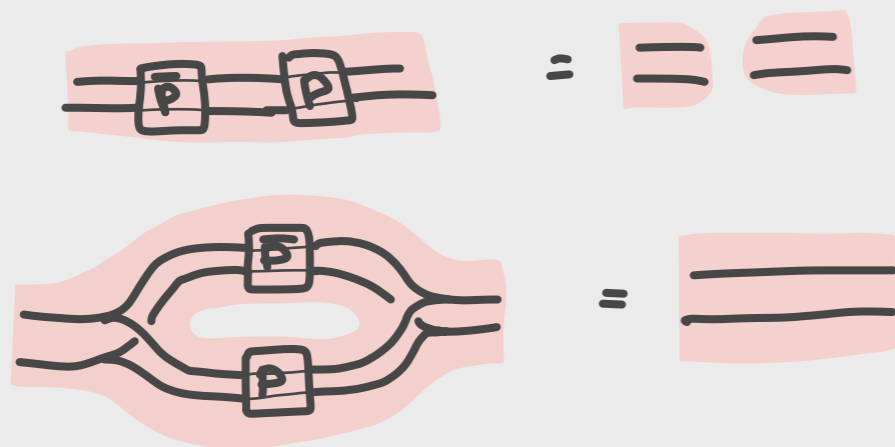
# Tape diagrams

- Predicates and Co-reflexives

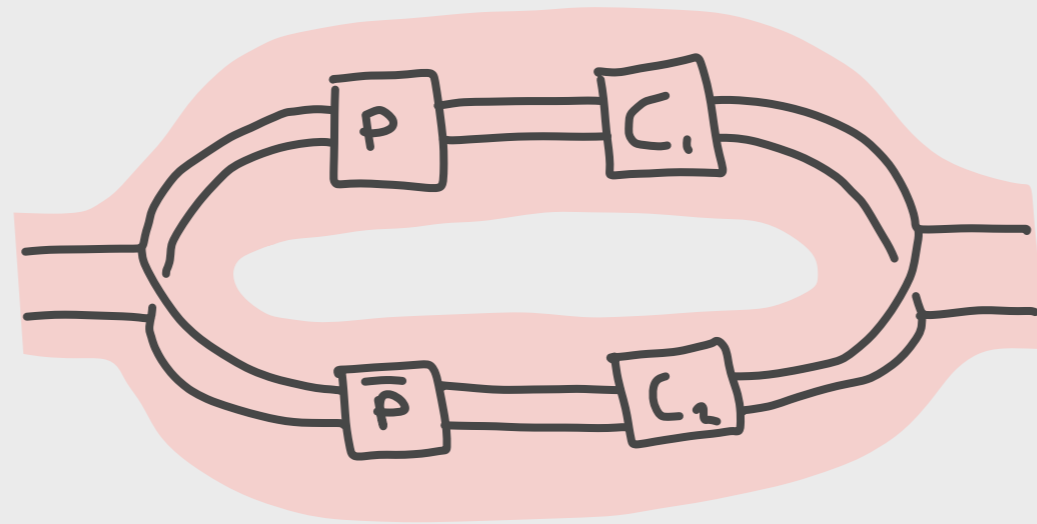
Given a Predicate  $P \subseteq X^2$



$\bar{P}$  (complement of  $P$ ) satisfies:



# Tape diagrams



"if  $p$  then  $c_1$ , else  $c_2$ "

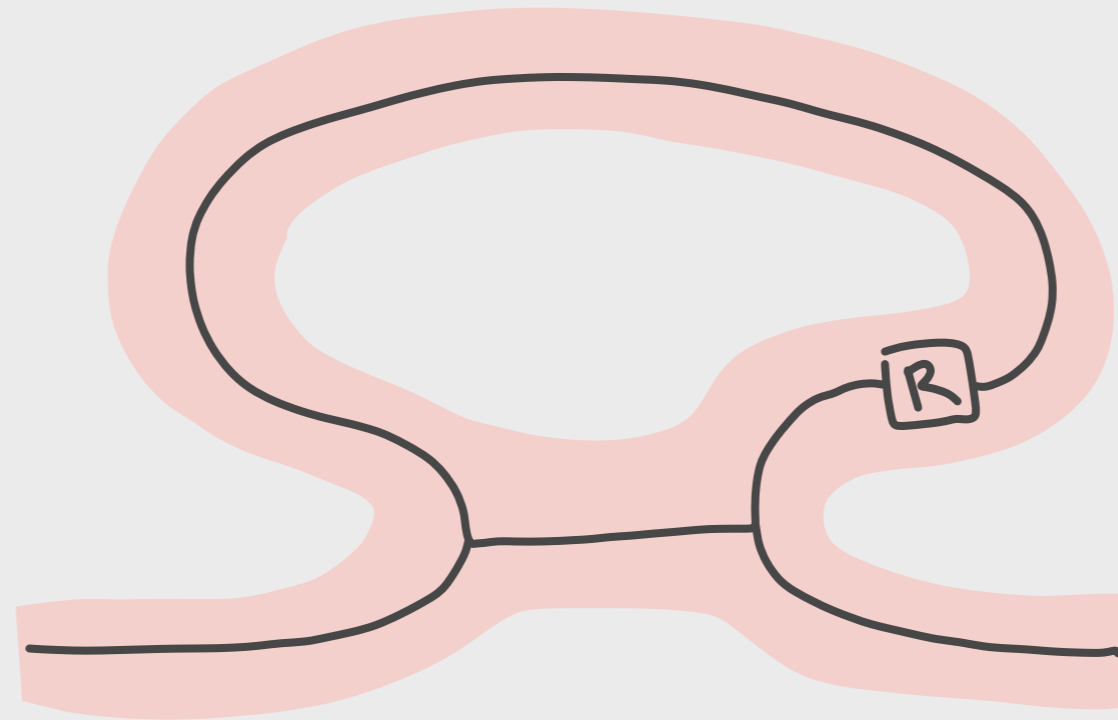
# Tape diagrams

- Iteration...



# Tape diagrams

- Iteration...

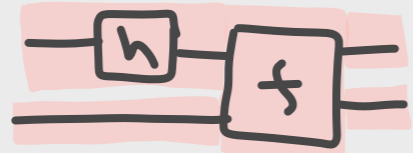


$R^*$

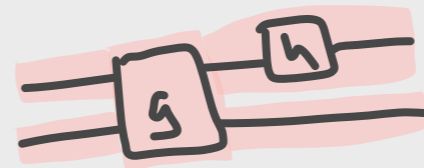
Reflexive  
Transitive  
Closure

# Tape diagrams

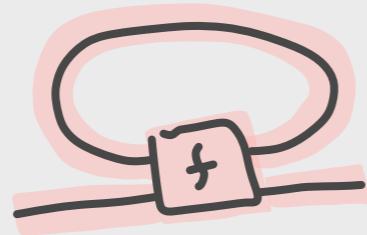
- Uniform trace...



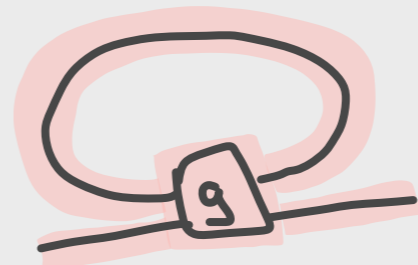
$\simeq$



$\Downarrow$



$\simeq$



# Tape diagrams

- Simple Program...

# Tape diagrams

- Simple Program...

$\{ x = A \wedge y = B \}$

while  $(x \neq y)$  do {

    if  $x > y$  then  $x := x - y$

    else  $y := y - x$

}

$\{ x = y = \text{gcd}(A, B) \}$

# Tape diagrams

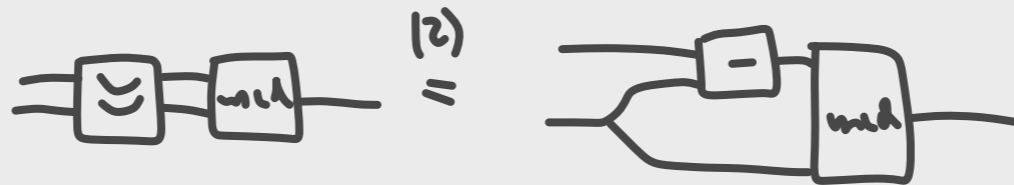
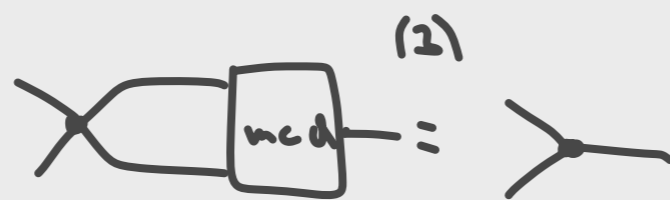
- Simple Program...

$$\text{gcd}(v, w) = \begin{cases} v & \text{if } v = w \\ \text{gcd}(v-w, w) & \text{if } v > w \\ \text{gcd}(v, w-v) & \text{if } v < w \end{cases}$$

# Tape diagrams

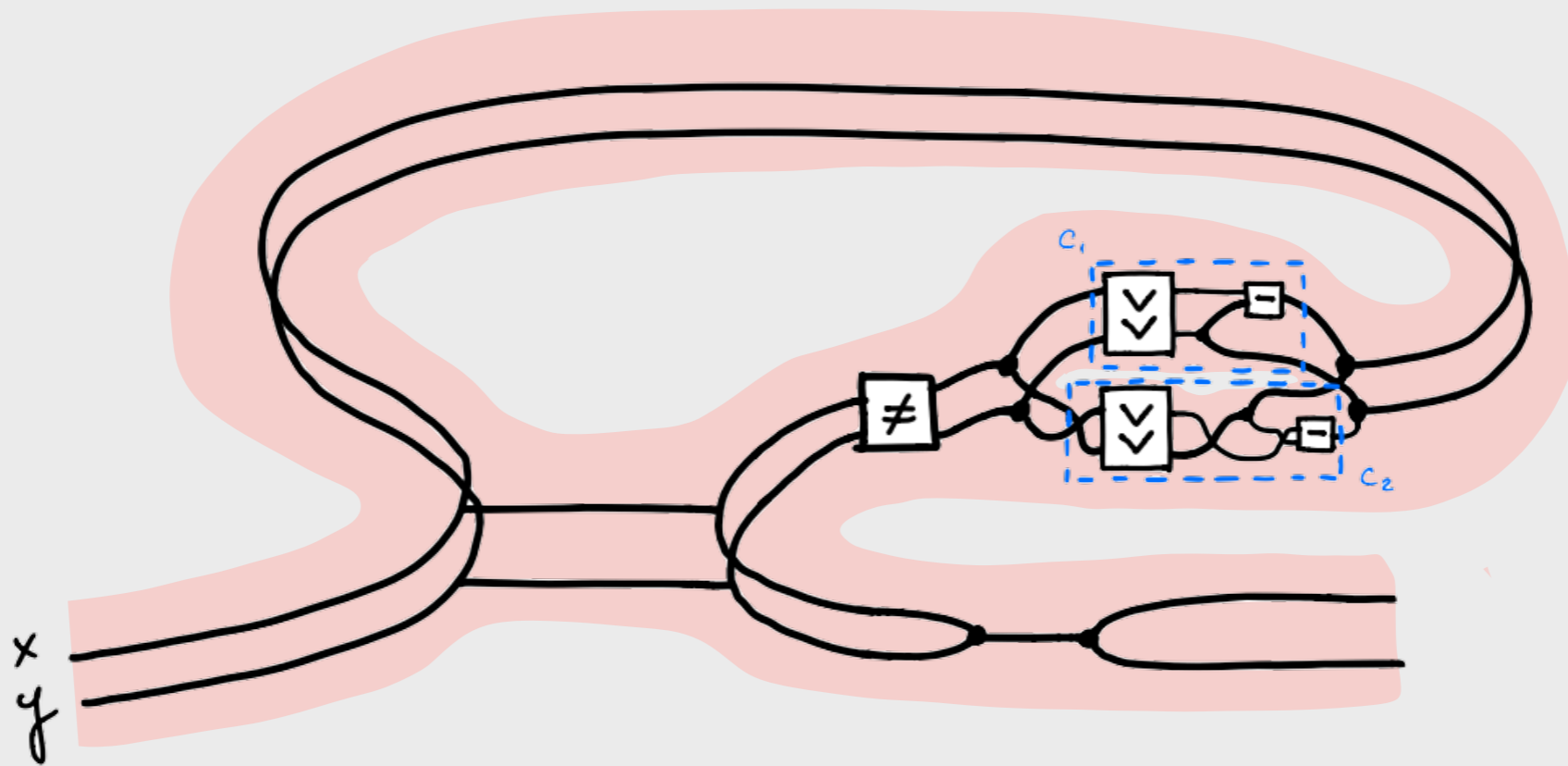
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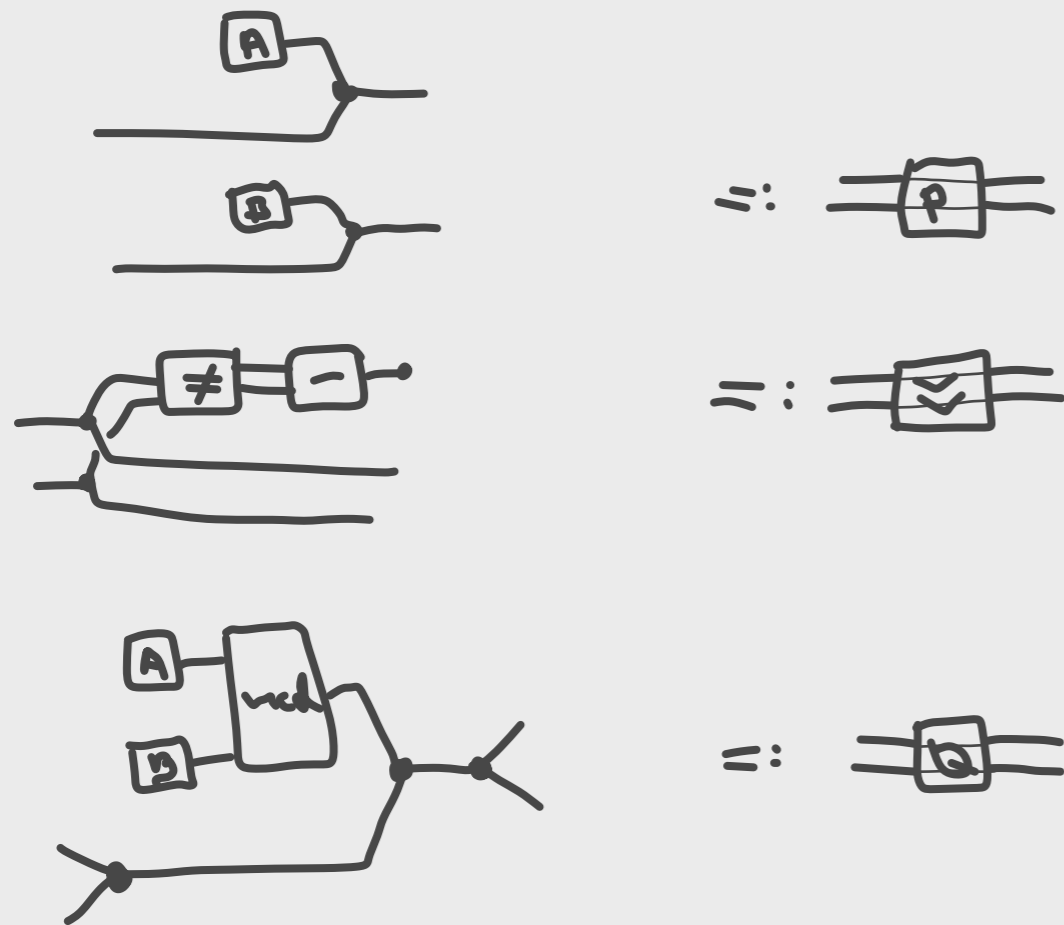
# Tape diagrams

- Euclid algorithm with tape diagrams



# Tape diagrams

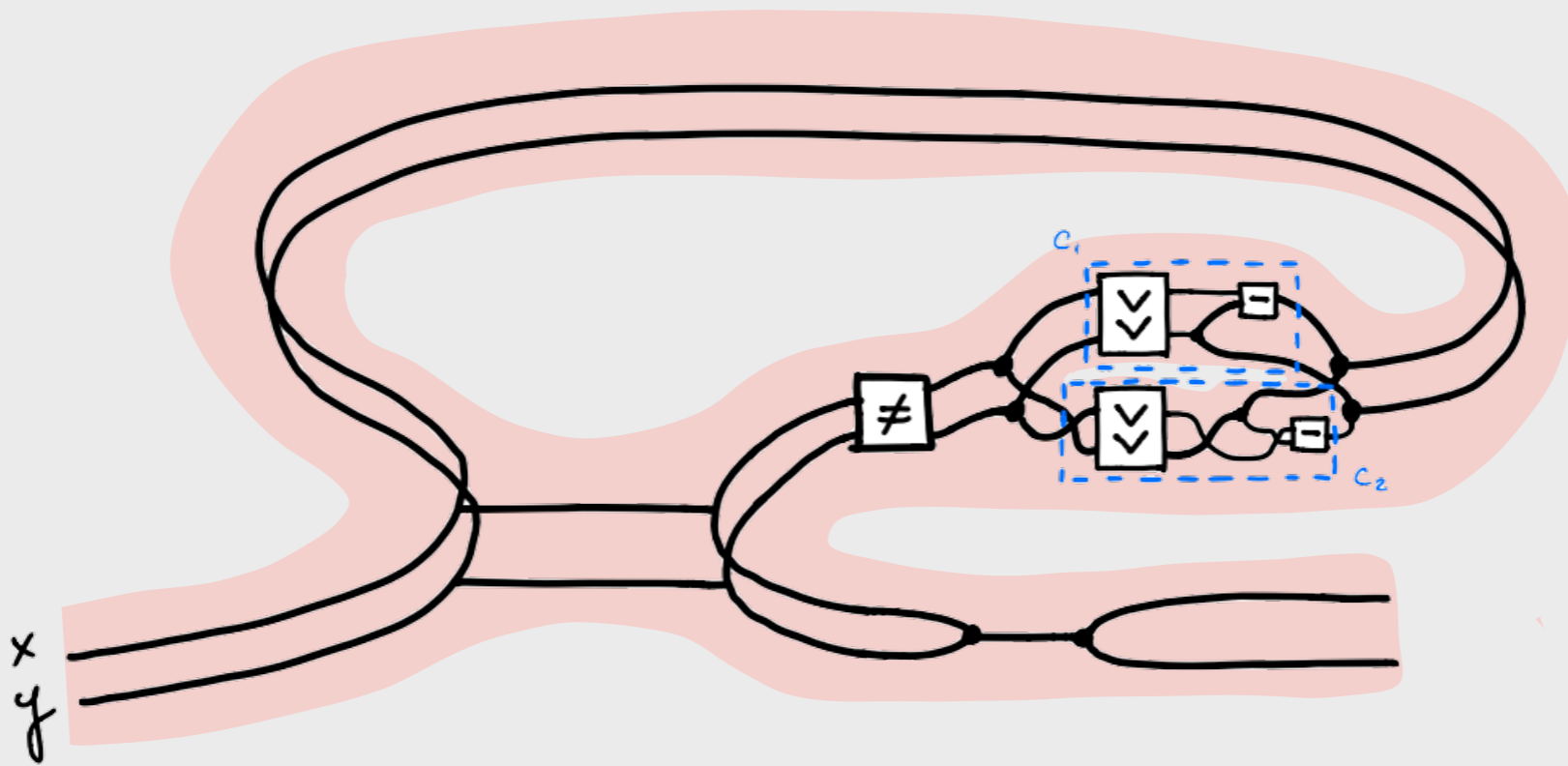
- Simple Program...





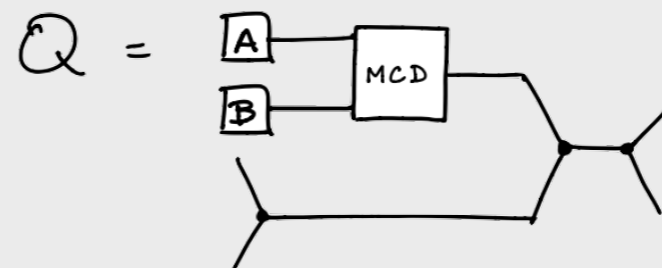
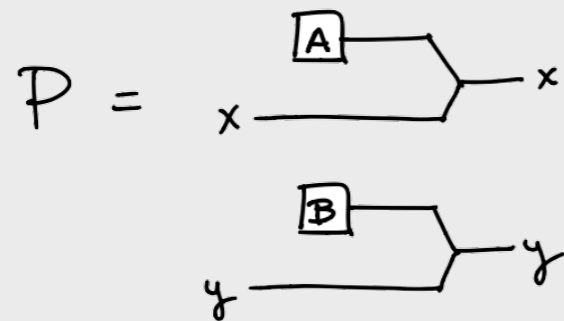
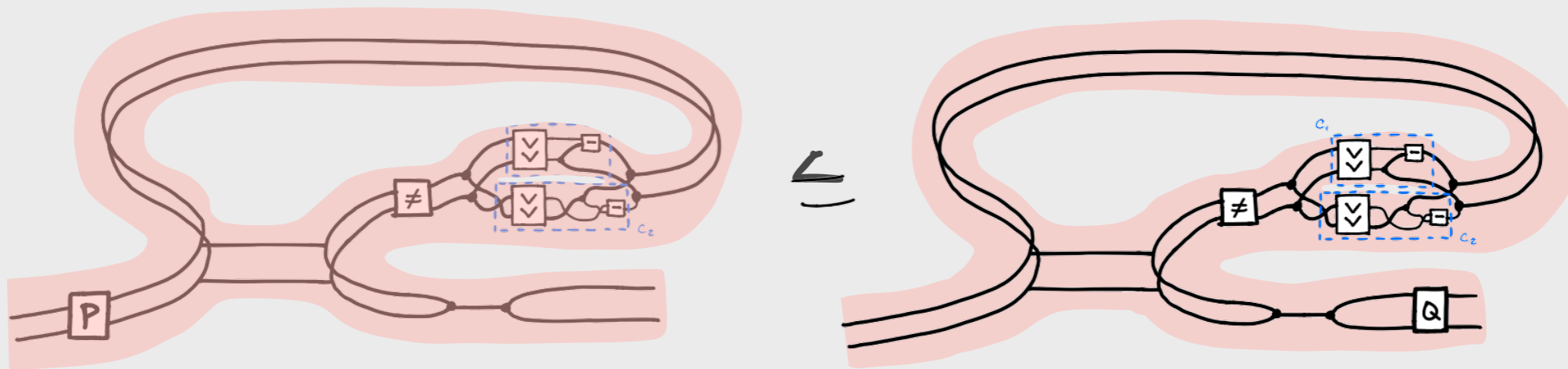
# Tape diagrams

- Simple Program...



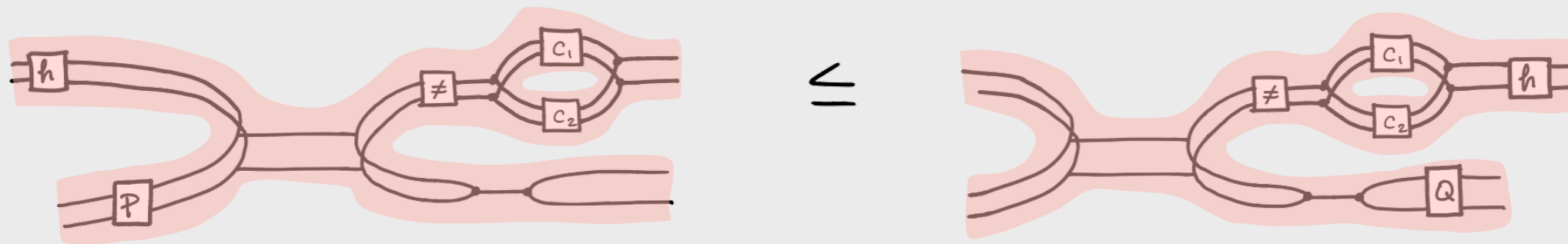
# Tape diagrams

- If we set  $a=x$  and  $y=b$  at the start of the program, we want to show  $\text{mcd}(a,b)=x$ .

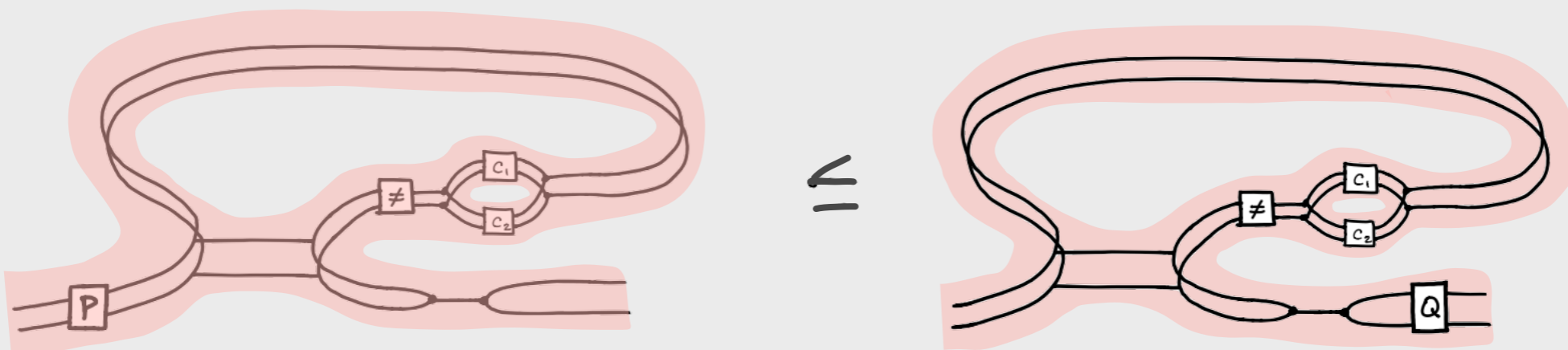


# Tape diagrams

- Proof strategy: If we can show

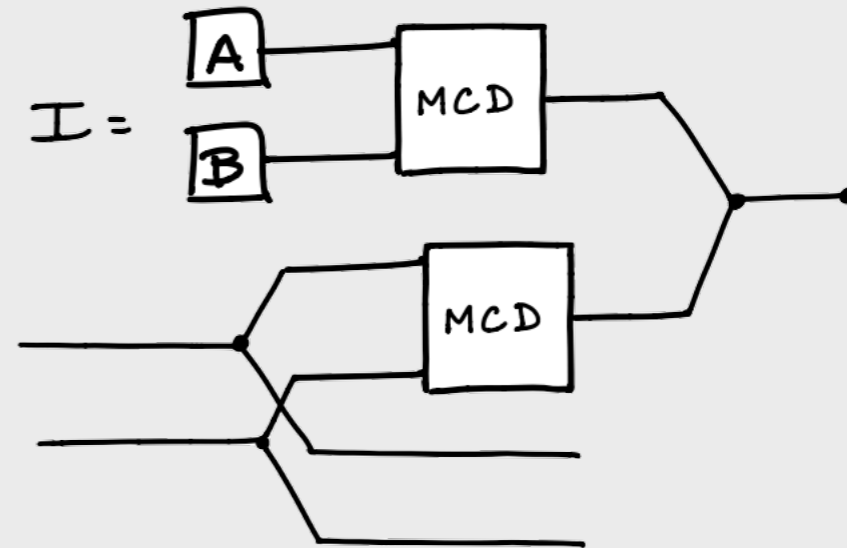


- For some  $h$ , then we can conclude:



# Tape diagrams

- Clever choice is I.
- Substitute I for P everywhere, okay since P less than I

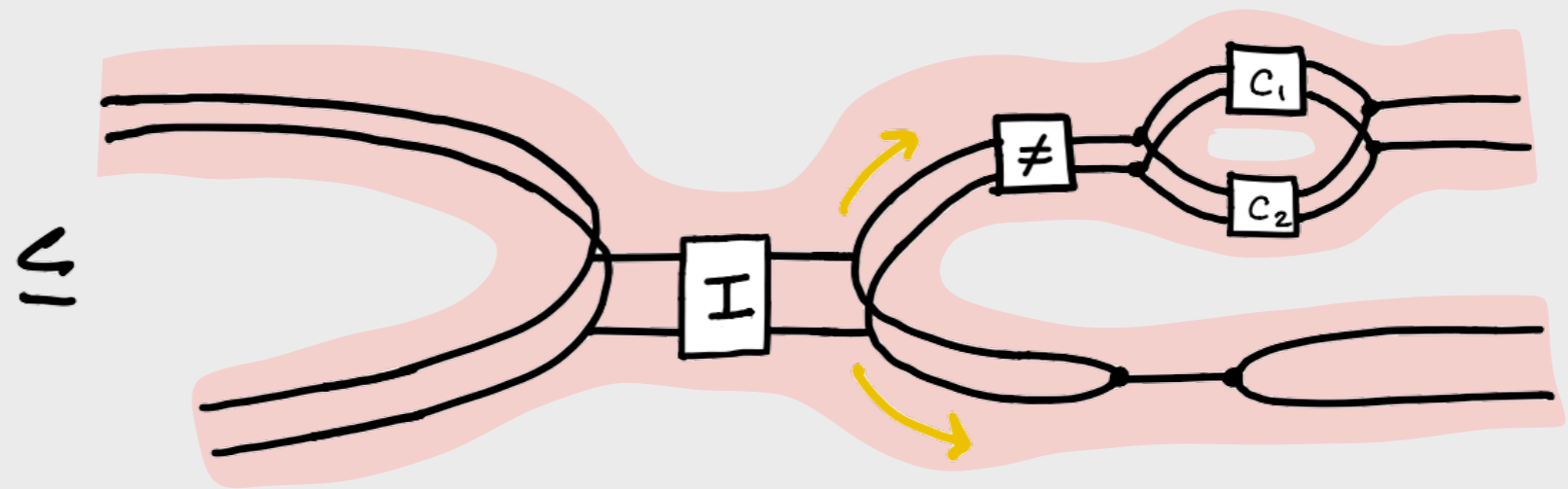
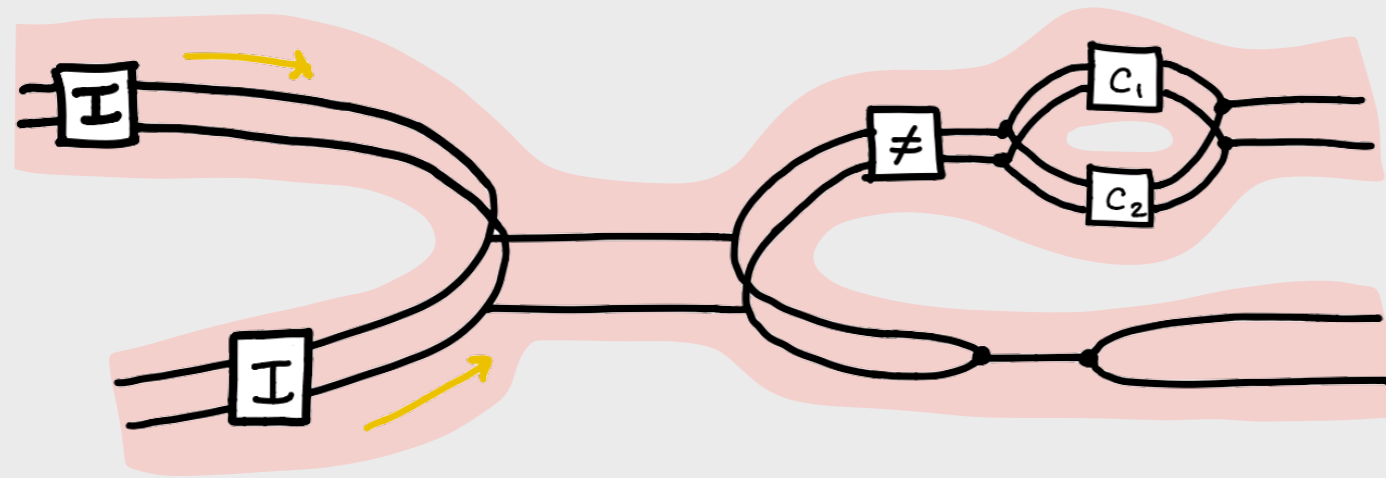


# Tape diagrams

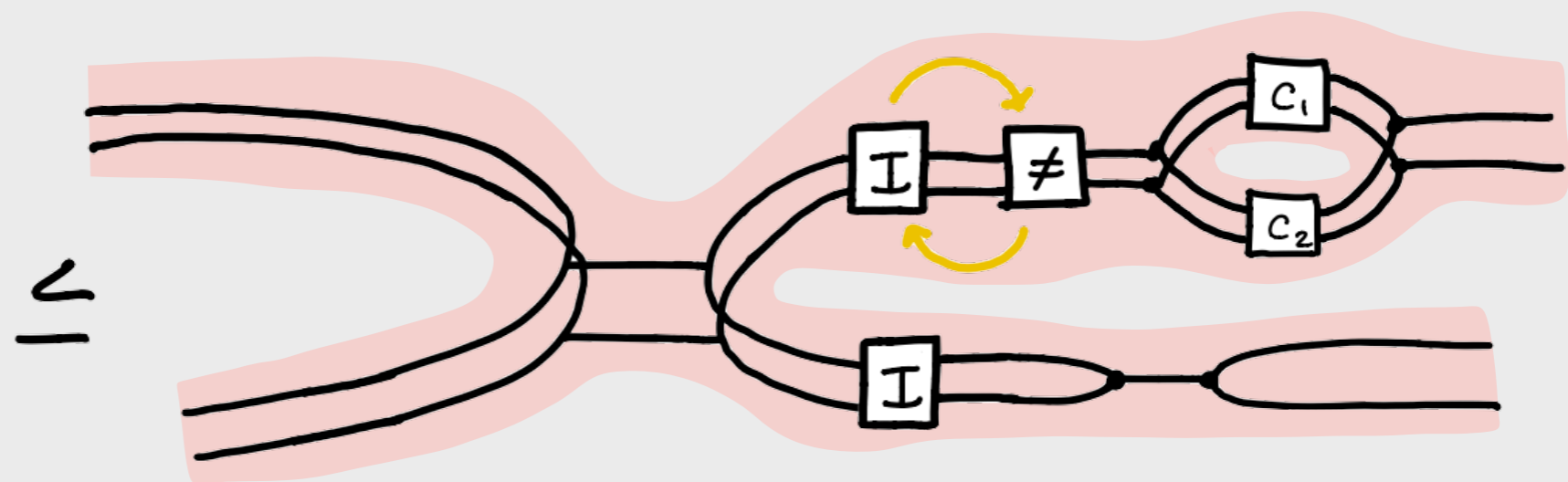
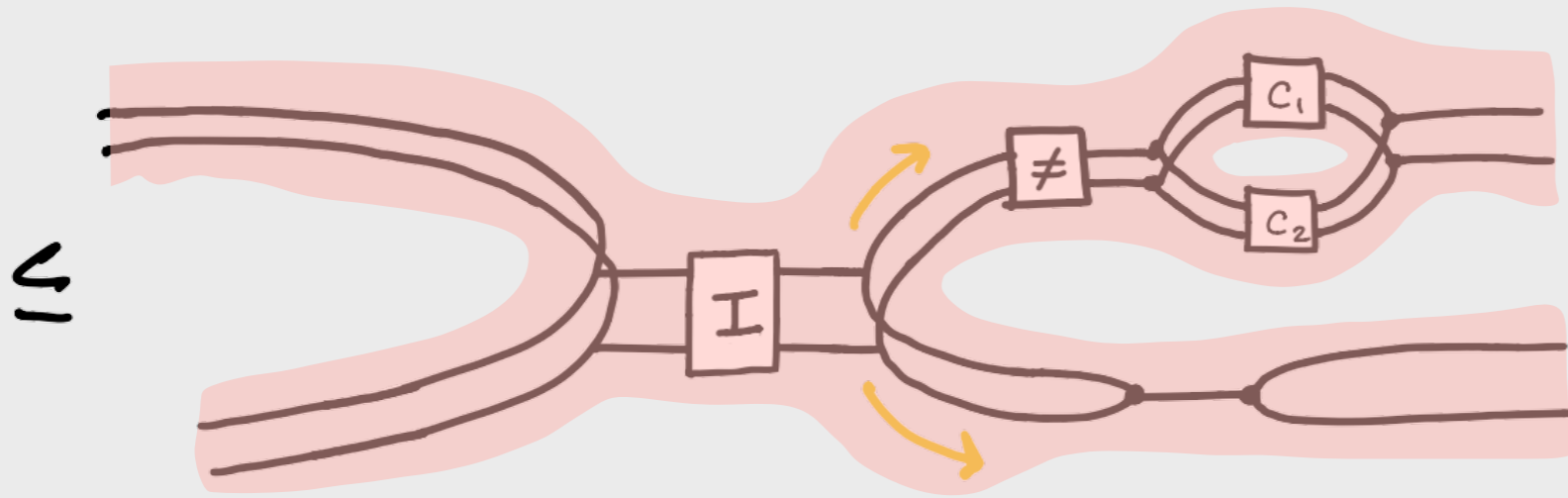
- New Formulation



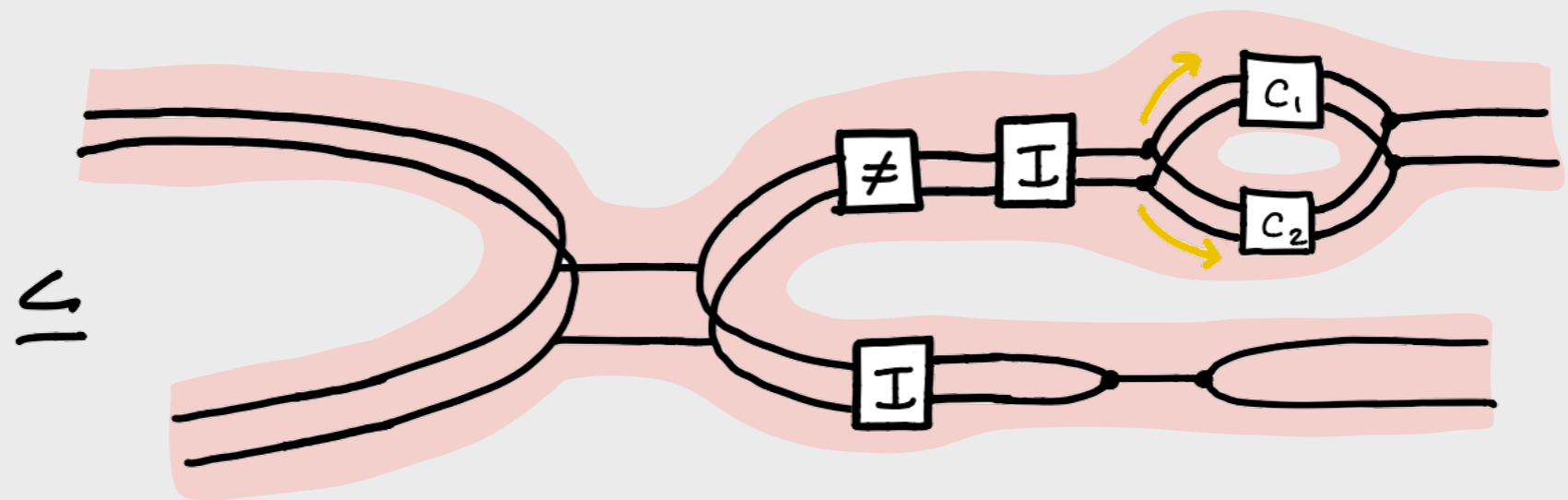
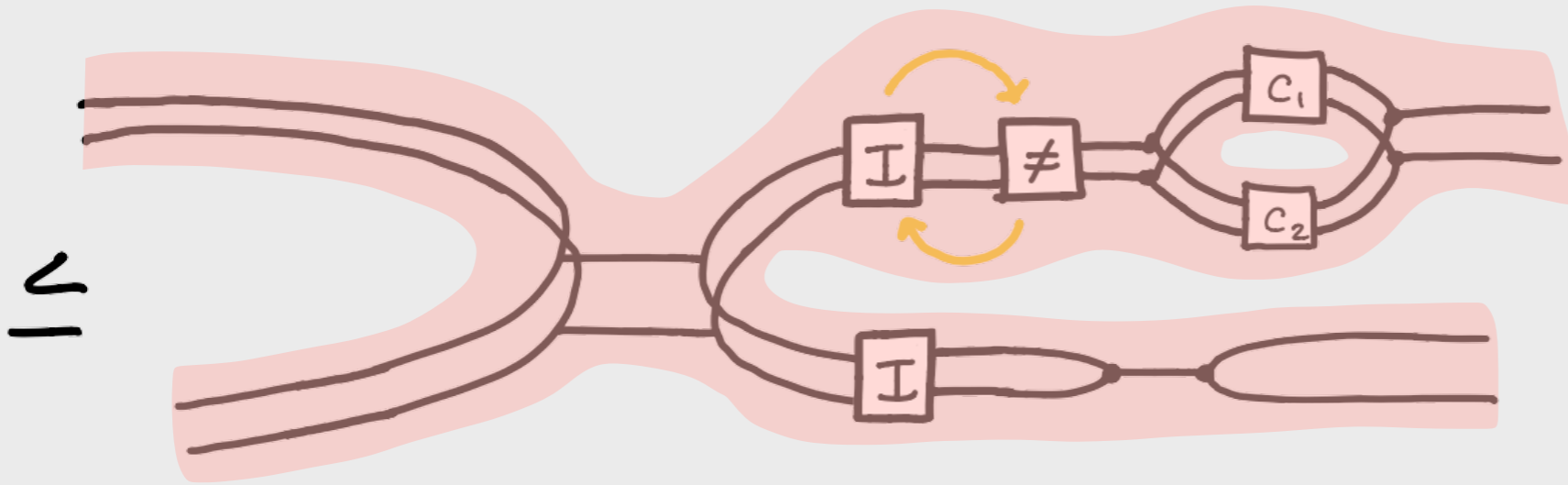
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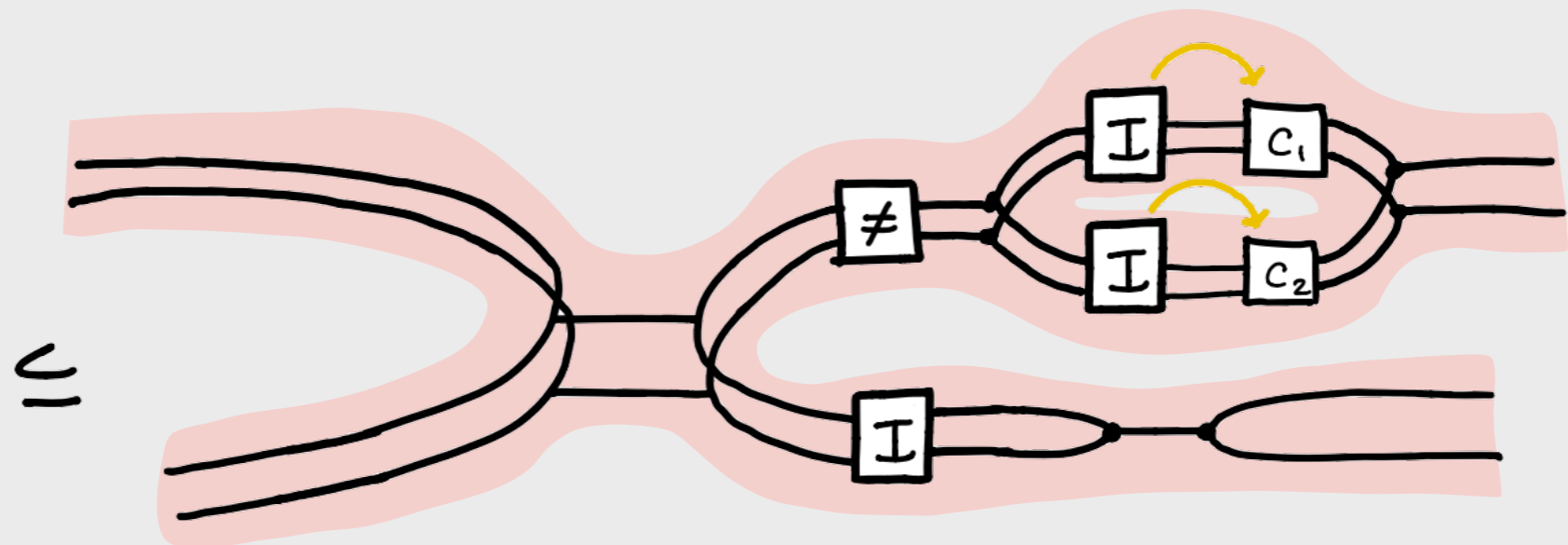
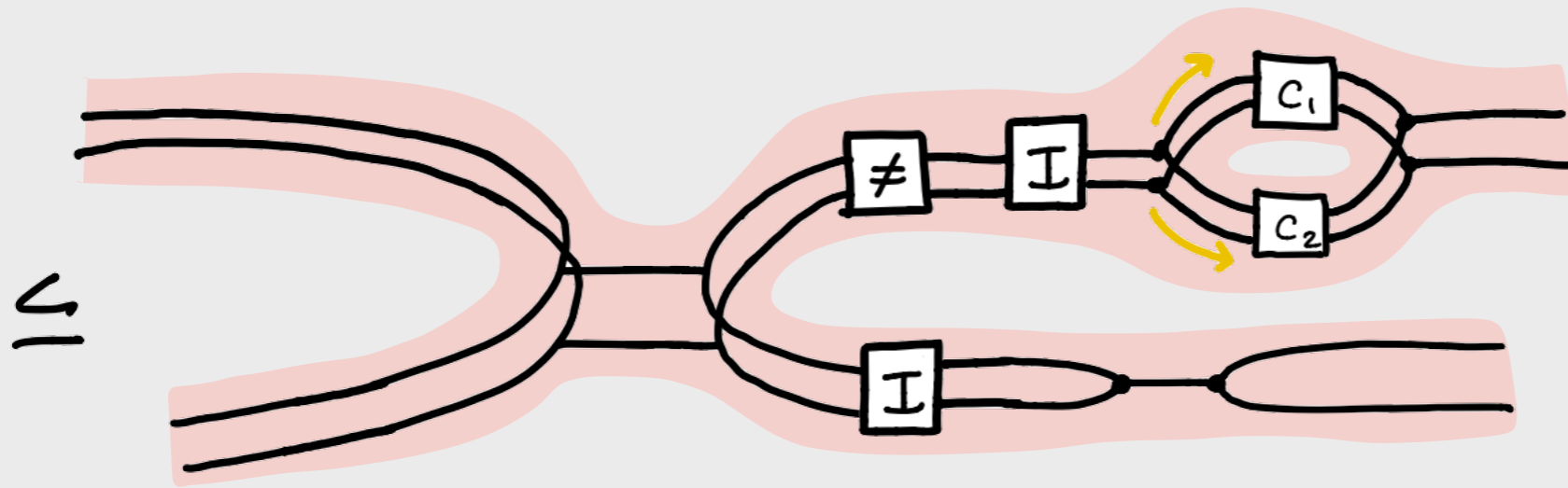


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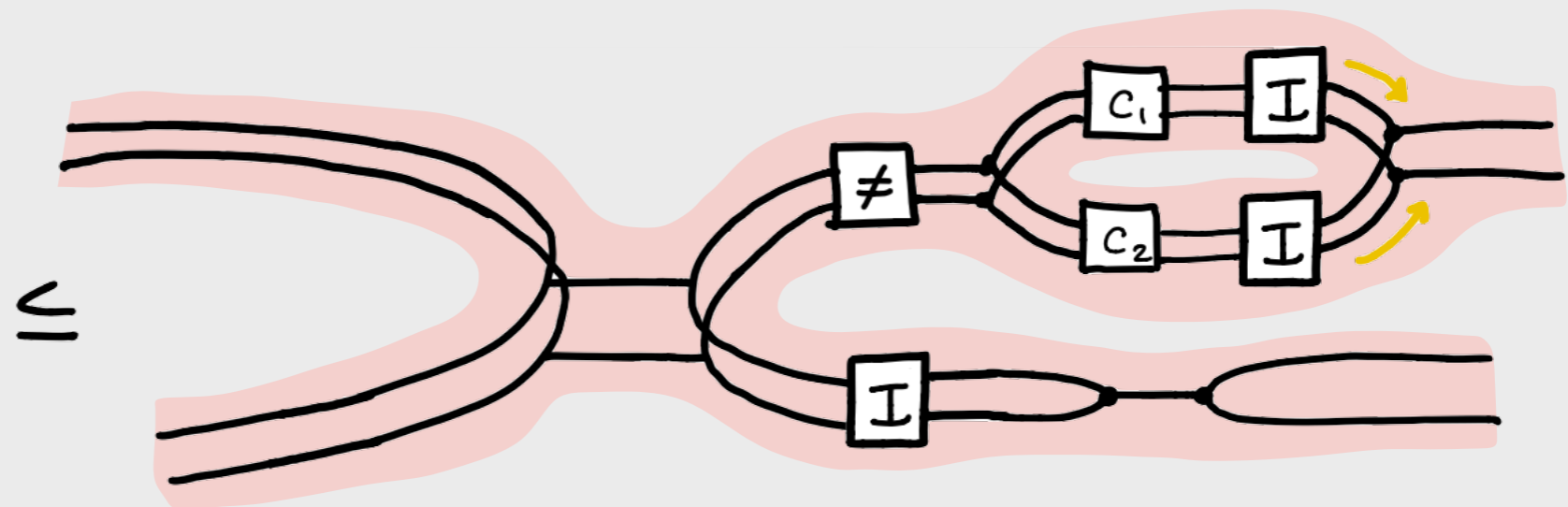
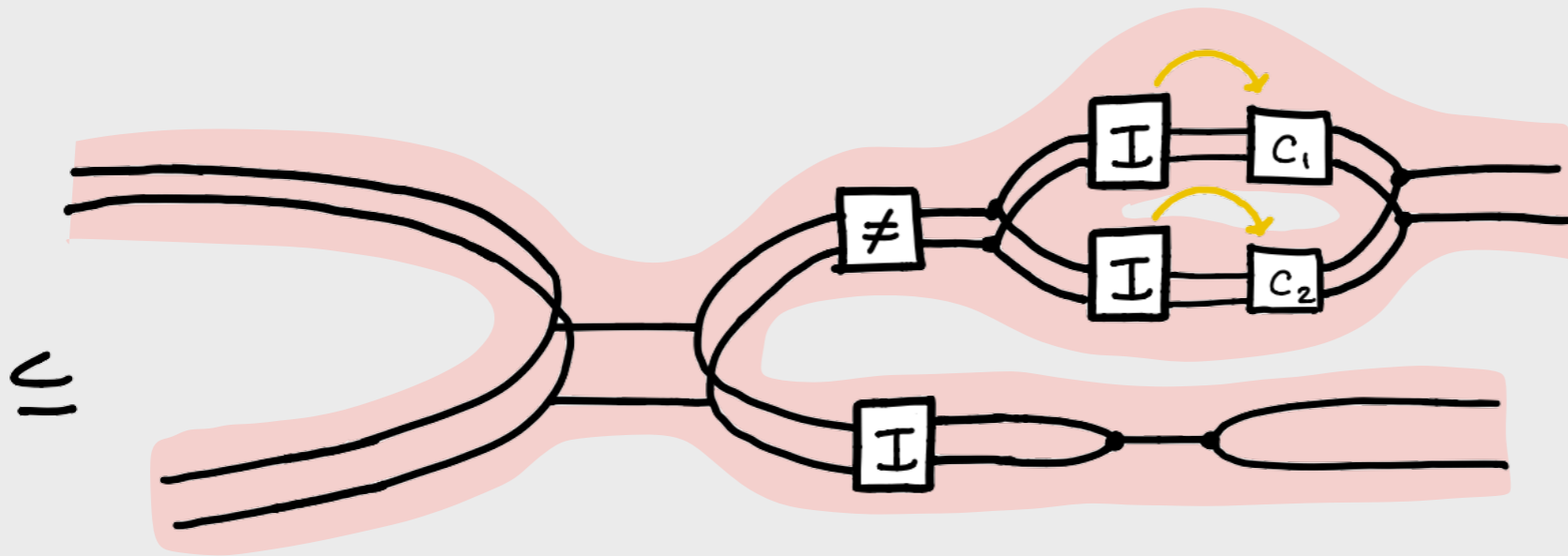




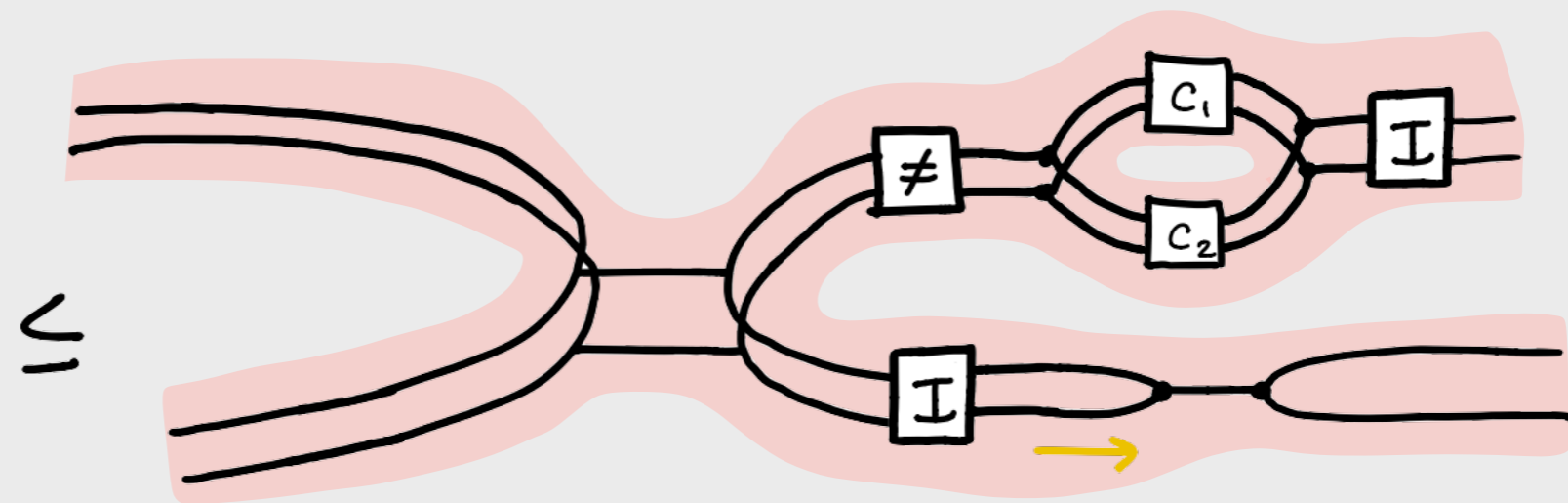
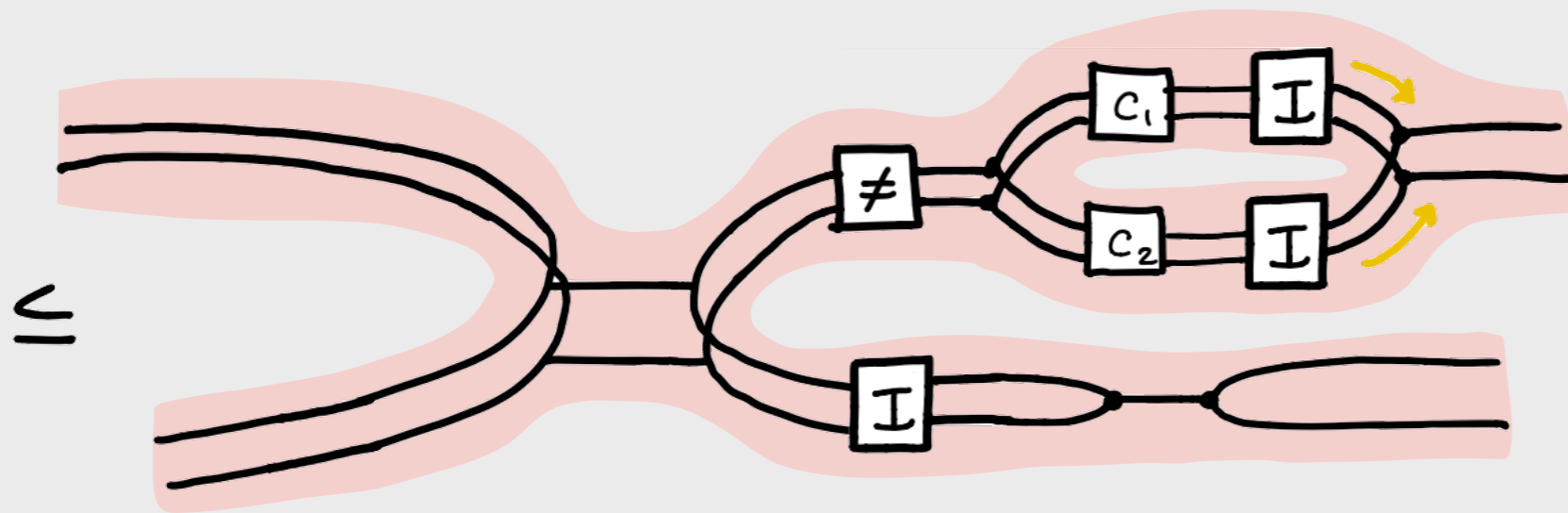
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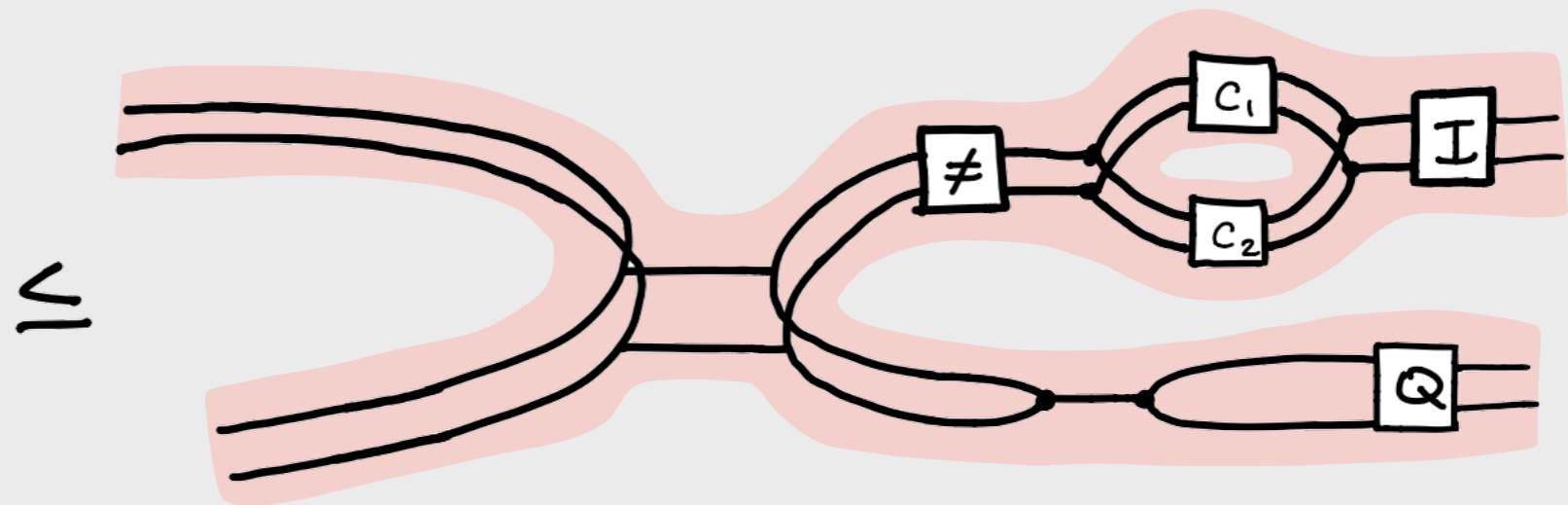
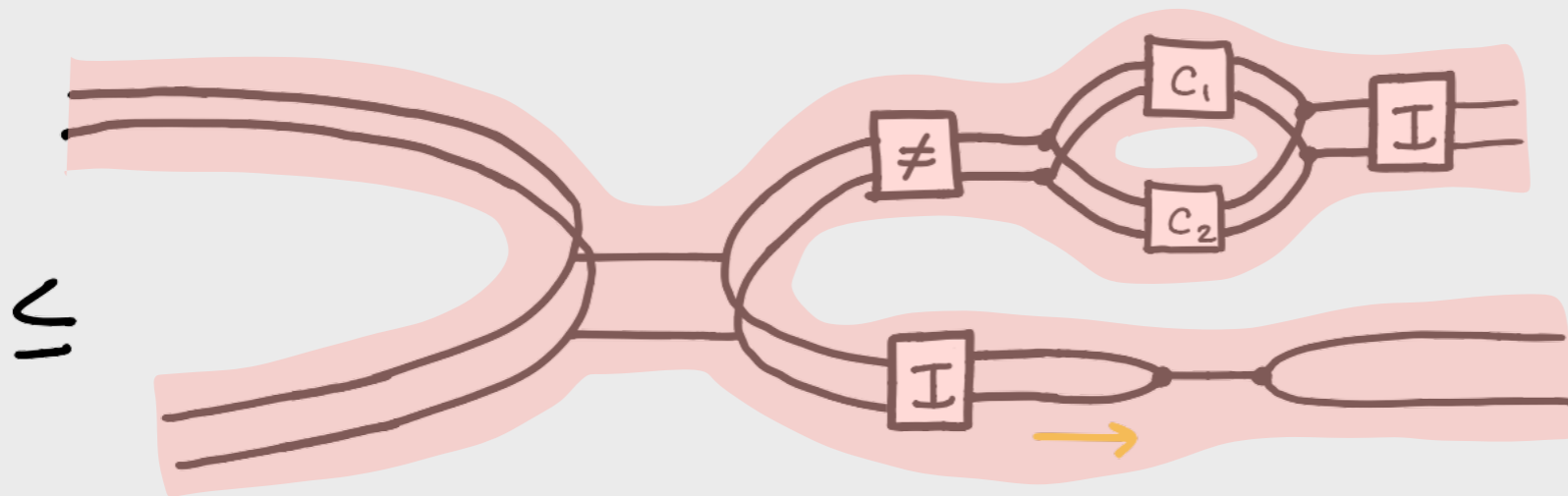
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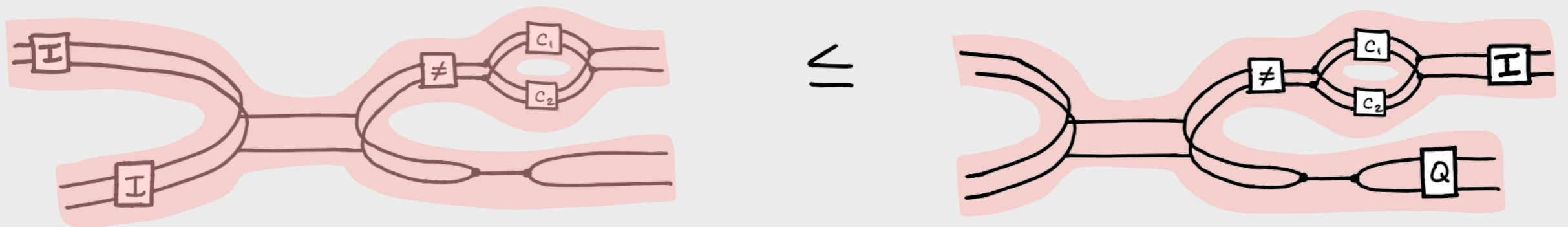
# Tape diagrams



# Tape diagrams



# Tape diagrams



# Future directions...

- Show universal property of  $S^*$

$$S^* = \nu x. x; \triangleright \vee 1.$$

- Link to distributive allegories? – maybe division allegories?
- Other models of type:  $\mathcal{T}_{\mathcal{E}, \mathcal{E}} \rightarrow \mathcal{REL}$

End